BREAKING THE COSMIC CODE: QUANTUM SPACES ARE REAL, NOT IMAGINARY STRUCTURES

GENE ALEXANDRESCU

Abstract. We discover the mathematical dual-space and use it to explain the physical nature of imaginary numbers and quantum spaces. We hypothesize that our universe is an overarching four-dimensional spacetime manifold, as Einstein believed, but with a secret twist. We show that quantum spaces, which we now think of as abstract mathematical structures, are in fact real spaces embedded in, but at the same time distinct from, the space of the universe. For this reason elementary particles are dimensionless mathematical points for us and we can only perceive their fields.

"The role of complex numbers in quantum theory had long struck me as a quite crucial one. If the correct geometry for the world is to be a closely quantum one, then these same complex numbers must be an essential part of this geometry. ...It had seemed fitting that this might be the geometry most basic to the structure of the physical world. Yet in its most obvious manifestations, physical geometry seems to be geometry over $\mathbb{R}$, not $\mathbb{C}$."

—Sir Roger Penrose [1]

I dedicate this work to Rafael Bombelli, who invented imaginary numbers. Without his "wild thought," there would be no concept of quantum spaces and much of today's physics would have been impossible.

Contents

2020 *Mathematics Subject Classification.* 11; 97F50.

Key words and phrases. Imaginary numbers, imaginary unit, twin communities, frames and observers, dual-space: "here" and "there," elementary particles, orthogonal spaces, Rafael Bombelli.

1. Introduction

In the field of mathematics, imaginary numbers have long been a subject of intrigue and fascination. Once considered as mathematical curiosities, these numbers have found remarkable applications in various branches of science and physics. One such domain where imaginary numbers play a crucial role is in the understanding of quantum spaces and the behavior of particles at the microscopic level.

Elementary particles, the fundamental constituents of all matter in this universe, exist within quantum spaces. They can only be described within quantum mechanics using complex mathematical formulations. They are not considered "imaginary" by scientists in the colloquial sense but are represented through abstract mathematical models because of their counter-intuitive properties at the quantum level. They are dimensionless and only their fields can be detected and measured. Until now, no one could provide an explanation for the physical nature of these spaces—as real matter in objective reality—or where exactly they exist within or outside the universe.

There exists a perceived dichotomy with a significant knowledge gap between the classical physics that explains observable matter and the quantum physics that governs elementary particles. This dichotomy does not imply a fundamental flaw in human perception, but rather a challenge that arises from attempting to reconcile our everyday experiences with the counterintuitive predictions of quantum mechanics.

We aim to address these complex issues by drawing upon the accumulated wisdom of brilliant mathematicians and physicists who have paved the way for us. A pivotal figure in this lineage is Rafael Bombelli, the genius scientist who invented imaginary numbers, an idea that seemed abstract initially but now plays a crucial role in our understanding of quantum mechanics.

1.1. **Rafael Bombelli.**

In 1572 Rafael Bombelli (1526-1572), an Italian mathematician and engineer, published "L'Algebra," a collection of three books on the theory of polynomials. He claimed that these books *would enable a beginner to master the subject* [2].

In the first book of "L'Algebra," he solved cubic equations by having the ingenious idea of imagining a new kind of numbers to represent $+\sqrt{-1}$ and $-\sqrt{-1}$ by naming them "piú di meno (pdm)" (plus of minus one) and "meno di meno (mdm)" (minus of minus one). He coined $\sqrt{-1}$ a "wild thought."

He believed that these numbers were independent of the real numbers and had special properties that the real numbers did not have.

But to this day no one is able to guess where this "wild thought" came from.

Over time, the two terms became the imaginary units $+\mathbf{i}$ and $-\mathbf{i}$ that we use in so many places today.

1.2. **René Descartes.**

Descartes was a French philosopher, mathematician and writer who, in 1637, wrote the paper, *La Géométrie*, in which he considered square roots of negative numbers a geometric impossibility, "fake artifacts of sloppy algebra."[3]

He coined them "imaginary numbers" with a clear derogatory intention and commented that "even though one can imagine that an equation has as many roots as its degree, there might not exist real numbers corresponding to all of the imagined roots."

Yet, despite the fact that Descartes used the word "imaginary" as an offensive term to dismiss square roots of -1, they survived over centuries, yet inconspicuously slowing down the scientific progress.

1.3. **Leonhard Euler.**

In the 18th century, the Swiss mathematician and physicist Leonhard Euler (1707-1783) worked in many areas of mathematics and physics and introduced several modern concepts and notations, including the letter $\mathbf{i}$ to denote the square root of -1. He also suggested that

$$\boxed{a + b\mathbf{i}} \tag{1.3.1}$$

is the general form of a complex number.

In 1777, Euler introduced the notation $\mathbf{i}$ and $-\mathbf{i}$ for the two square roots of -1. He also introduced the notation $a + b\mathbf{i}$ for a general complex number. Subsequently, Euler began to study the behavior of functions other than polynomials when given complex-valued arguments, for example, the exponential function.

1.4. **Carl Friedrich Gauss.**

Two centuries later, Carl Friedrich Gauss (1777-1855), a German mathematician, contributed extensively to several fields of science. Considered one of the *greatest mathematicians since antiquity*, Gauss had a significant impact on mathematics, physics, and astronomy and is considered one of the most influential mathematicians [4].

Gauss clarified the concept of "complex numbers" but criticized this unfortunate choice:

> "*That this subject [of imaginary magnitudes] has hitherto been considered from the wrong point of view and surrounded by a mysterious obscurity, [it] is to be attributed largely to an ill-adapted notation. If, for example,* $+1, -1, \sqrt{-1}$ *had been called direct, inverse and lateral units, instead of positive, negative and imaginary (or even impossible), such an obscurity would have been out of the question.*"

However, he could not convince his pairs to dismiss the troublesome term "imaginary," and he was not alone in trying to change that.

1.5. **William Rowan Hamilton.**

The scientist who made the greatest contribution to demystifying the enigmatic concept of "imaginary numbers" was William Rowan Hamilton (1805-1865), an Irish mathematician, physicist, and astronomer who introduced new mathematical concepts and techniques into classical mechanics, electromagnetism, and optics. In pure mathematics, he is best known for the discovery of quaternions, the algebraic tool used in quantum mechanics.

Hamilton wanted to eliminate the difficulties of working with imaginary quantities by extending the concept of "real numbers" to a larger set of numbers, which he called "algebraic couples" and wrote (a, b) in which both a and b are real numbers but do not belong to the same community.

In the introduction to his book "Theory of Conjugate Functions, or Algebraic Couples" Hamilton wrote [5]:

> "*The* Theory of Conjugate Functions *gives reality and meaning to conceptions that were before* Imaginary, Impossible, or Contradictory*. . . The author acknowledges with pleasure that he agrees with M. Cauchy, in considering every (so-called)* Imaginary Equation *as a symbolic representation of two separate Real Equations.*"

Hamilton used the euphemistic expression "possible extraction" . . . from this reality:

> "*In the theory of single numbers, the symbol* $\sqrt{-1}$ *is* absurd*, and denotes an* impossible extraction*, or a merely* imaginary number*; but in the* theory of couples*, the same symbol* $\sqrt{-1}$ *is significant, and denotes a* possible extraction*, or a* real couple*. . .* "

The advantage of the construction of complex numbers as algebraic couples, however, is that the imaginary concept is replaced by the notion of ordered pairs of real numbers, which already have a meaning in physical reality.

1.6. **Benjamin Peirce.**

In an article published in 1874, Benjamin Peirce brings an interesting approach [6]:

> "*When the formulas admit of intelligible interpretation, they are accessions to knowledge. . . But the most noted instance is the symbol* $\sqrt{-1}$*, called the "impossible" or "imaginary," and which. . . may be more definitely distinguished as the symbol of "semi-inversion"... The strongest use of the symbol is to be found in its "magical power of doubling the actual universe, and placing by its side an ideal universe, its exact counterpart, with which it can be compared and contrasted," and, by means of curiously connecting fibres, form with it an organic whole, from which modern analysis has developed her surpassing geometry.*"

Similar to other great mathematicians who thought that the "imaginary" concept should be replaced by another, "more real" concept, Peirce advocates the possibility of an "ideal universe" which he places next to the real universe and considers its "exact counterpart."

2. On the nature of Imaginary Unit

By definition, a complex number $z = a + b\mathbf{i}$ has three components: two numbers a and b belonging to the set of real numbers $\mathbb{R}$, and the imaginary unit $\mathbf{i}$. The only difference between a and b is their numerical value.

In the complex number z, a is the real part and b in $b\mathbf{i}$ is the imaginary part. By convention, the imaginary part does not contain the imaginary unit $\mathbf{i}$.

Multiplying b by $\mathbf{i}$ (strictly speaking, by $+\mathbf{i}$) moves it to the axis $\mathfrak{Im}$ and causes it to be called the "imaginary part," as shown in Figure 1 on page 5. Even more interestingly, reversing the operation by multiplying $b\mathbf{i}$ by $-\mathbf{i}$ returns it to the real axis: $b\mathbf{i} \cdot -\mathbf{i} = b$.

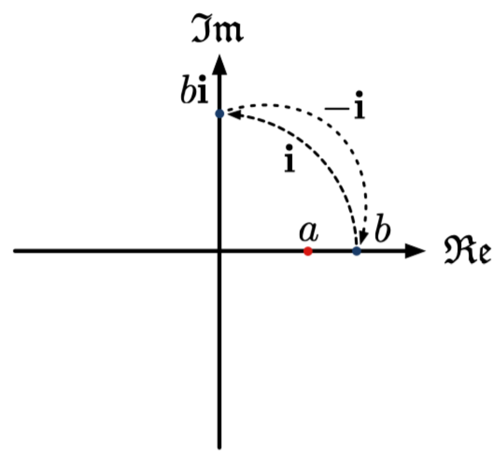

Figure 1. Numbers a and b are always real.

The debate about the nature of "imaginary" numbers arose at the same time as the numbers themselves. The fact that multiplying a number belonging to the real space $\mathfrak{Re}$ by $\mathbf{i}$ moves it into the imaginary space $\mathfrak{Im}$ certainly seems strange at first.

But the very nature of b—belonging to the set of real numbers $\mathbb{R}$—has not changed while it is on the imaginary axis. If we simply assert that the number has changed the "community" to which it belongs, this concept immediately becomes more understandable and acceptable to our minds: the imaginary unit $\mathbf{i}$ has just caused b to move from one axis to the other.

Indeed, if we consider each of the two axes as a one-dimensional space, then $+\mathbf{i}$ moves b from one one-dimensional real space to another one-dimensional real space—which we call "imaginary space" just to distinguish it from the first space. Peirce called it "an ideal universe" [6].

The imaginary unit $\mathbf{i}$ does not just move a number from $\mathfrak{Re}$ to $\mathfrak{Im}$ and back. In a complex plane, $\mathbf{i}$ has two geometric functions: (1) it represents the imaginary unit of the $\mathfrak{Im}$-axis [7], and (2) each successive multiplication of b by $\mathbf{i}$ produces a 90° counterclockwise rotation (left quarter turn), while $-\mathbf{i}$ produces a 90° clockwise rotation (right quarter turn). (See Figure 2.)

The effect of two left or right rotations is the reversal of direction, i.e., an U rotation. The statement $\mathbf{i}^2 = -1$ is merely the algebraic version of the geometric fact that the sum of two quarter turns $90° + 90°$ is an even 180° angle.

This second function of the imaginary unit **i** is commonly known as **i**-*operator* (or **j**-*operator* in electrical engineering).

3. Twin Communities

Note that so far this discussion has focused exclusively on the integer powers of **i** in order to better identify the "twin communities" encompassed by the complex numbers.

Interestingly, both axes of the plane used for algebraic couples are as real as the axes of the plane used for ordered pairs. In this way, the first function of **i**, which is the imaginary unit of the $\mathfrak{Im}$-axis, disappears: there is no longer an imaginary axis, since both axes are real. This fact is well known in physics and engineering, where complex algebra is routinely used for many mathematical operations in real spaces. This implies that the **i**-*operator* does not have much to do with the physical nature of the two spaces.

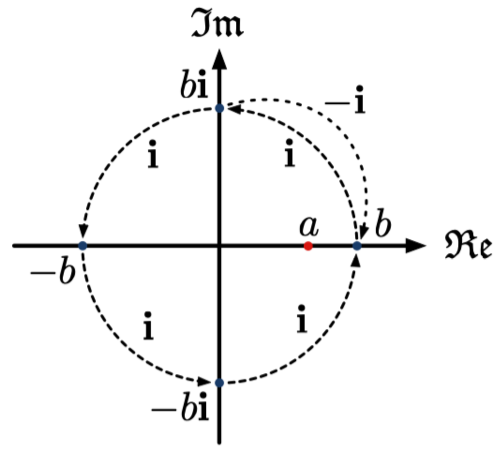

Figure 2. Multiplication of a number by **i** or -**i**.

On the other hand, if we consider the plane itself as a whole, it turns out that it is similar to the real 2D space, but somehow different from it. For this reason, the complex plane (a, b) is said to be isomorphic to (look exactly like) the real plane (a, b).

Hamilton's idea of representing complex numbers by algebraic couples is a good start: it establishes that the complex plane is a coordinate system with two dimensions, both of which can be called "real." The relationship between the two axes alone distinguishes the complex plane from the Cartesian plane, i.e., one dimension appears "imaginary" with respect to the other.

For physics, this conclusion means that we could stop thinking of quantum spaces as imaginary and view them as real entities, but with the caveat that we need to know what this imaginary relation means physically and not just mathematically.

To do this, we first need to find out what physical configuration of space maintains isomorphism with the Cartesian plane while encompassing two real and independent spaces. To do this, we need to discuss two well-known concepts in physics, but not previously associated with the notion of complex spaces.

4. Frames and Observers

The first is the concept of "reference frames" (or "frames of reference"), which arose from the need to observe phenomena from defined points of view, done either by humans or by measuring devices, collectively called "observers."

In mathematics, for example, the Cartesian plane is an "abstract 2D coordinate system."

In physics, "coordinate systems" remain abstract, but are associated with "reference frames," i.e., sets of physical reference points that uniquely locate and orient them. In this way, the physical "Cartesian reference frame" is contained in a mathematical "Cartesian plane."

The second point is the "concept of the observer $\mathbb{O}$."

We tend to think that definitions in mathematics are absolute and do not need an observer [8], e.g., "a real number" is "a number that is either rational or the limit of a sequence of rational numbers" [9].

However, in several branches of physics, such as special and general relativity and quantum mechanics, the role of the observer is mandatory.

Special relativity, for example, is based entirely on the concept of "observational frames of reference," that are in relative inertial motion [10]. Basically, in each frame of reference there is either an observer at rest or the observed phenomenon or phenomena.

In quantum mechanics, the role of the observer includes not only the measurement, but even the question of the observed phenomena. As the physicist Fritjof Capra explains [11]:

> "*The crucial feature of atomic physics is that the human observer is not only necessary to observe the properties of an object, but is necessary even to define these properties... This can be illustrated with the simple case of a subatomic particle. When observing such a particle, one may choose to measure—among other quantities—the particle's position and its momentum.*"

The seemingly different nature of its two coordinates, labeled "real" $\mathfrak{Re}$ and "imaginary" $\mathfrak{Im}$, suggests that the 2D complex plane could be interpreted as a union of two independent and real 1D vector spaces $\mathbb{S}_1$ and $\mathbb{S}_2$, rather than one real and one imaginary.

These spaces are real but orthogonal to each other, which means that they do not "see," i.e. interact or influence each other. Simply put, orthogonality of independent spaces occurs when the projection of one space onto the other is zero. For this reason, we call them "orthogonal spaces."

To avoid confusion between the colloquial use of the term "imaginary" and its mathematical definition, we use the concept of the observer as understood in physics, particularly in quantum physics, where the observer is the entity making a measurement. Consequently, we proceed to the next step by replacing the somehow questionable terms "real" and "imaginary" by two observer-related references.

5. The mathematical dual-space: "here" and "there"

We imagine a thought experiment in which there are two independent spaces $\mathbb{S}_1$ and $\mathbb{S}_2$, such as New York and Melbourne.

We begin our inquiry by imagining that in the space $\mathbb{S}_1$ there are two observers $\mathbb{O}_1$ and $\mathbb{O}_2$ observing two objects a and b, as shown in Figure 1 on page 5:

$$z_0 = a + b \mid a, b \in \mathbb{R} \tag{5.0.1}$$

From the point of view of the two observers inhabiting it, the space $\mathbb{S}_1$ is real, so it can be defined as a space of axis $\mathfrak{Re}$. The space $\mathbb{S}_2$ is not directly experienced by the two observers, so it can be considered as the space of axis $\mathfrak{Im}$.

At this time, we call the space $\mathbb{S}_1$ of the two observers $\mathbb{O}_1$ and $\mathbb{O}_2$ "here in $\mathbb{S}_1$."

When the two observers are in $\mathbb{S}_1$, the space $\mathbb{S}_2$ is distant for them. It is also real, but different and distinct from the observers' space. Since the observers are not in this space, it is not directly accessible to them—it is invisible to them—and therefore they might consider it "imaginary." For this reason, we call it "there in $\mathbb{S}_2$."

By definition, the observers $\mathbb{O}_1$ and $\mathbb{O}_2$, and the two objects a and b can travel in both spaces.

Let us redraw Figure 2 on page 6 with the set of the two orthogonal and real 1D spaces. As a reminder, we also keep the notations as complex spaces.

We start the experiment by deciding to send b into the space $\mathbb{S}_2$. To do this, we recall that $\mathbf{i}$ has two properties (1) to move a number from one axis to the other—in fact from one space to the other—and (2) to stick to the number b after the move, as a flag marking its position on either of the two $\mathbb{S}_2$ semi-axes.

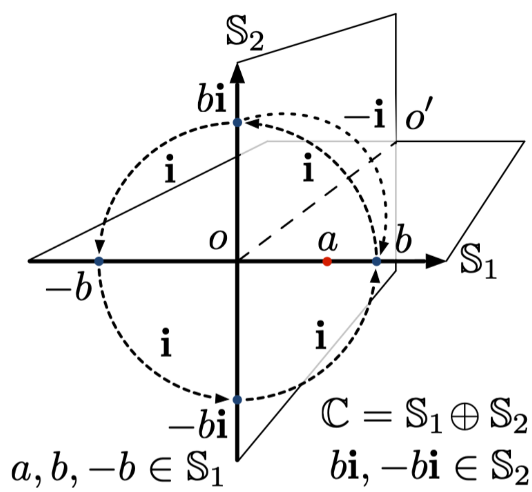

FIGURE 3. The complex set of two orthogonal 1D real spaces.

To do this, we multiply b by $\mathbf{i}$ so that it becomes $b\mathbf{i}$ in $\mathbb{S}_2$. At this moment there is a number in each space, so instead of the number $z_0 = a + b$ the new number is:

$$z_1 = a + b\mathbf{i} \mid a, b \in \mathbb{R} \tag{5.0.2}$$

The numbers a and b are still real, but a is "here" in $\mathbb{S}_1$ and b is "there" in $\mathbb{S}_2$, being multiplied by $\mathbf{i}$.

Step two: while the observer $\mathbb{O}_1$ is still in $\mathbb{S}_1$ and observes z_1 as usual, the observer $\mathbb{O}_2$ follows b in $\mathbb{S}_2$. Mathematically, moving to $\mathbb{S}_2$, the observer's $\mathbb{O}_2$ perspective changes from

z_1 to z_2, since she is also multiplied by $\mathbf{i}$:

$$z_2 = z_1 \cdot \mathbf{i} = a\mathbf{i} - b \tag{5.0.3}$$

Thus a becomes $a\mathbf{i}$ for the observer $\mathbb{O}_2$ and is now imaginary for her because it remained in $\mathbb{S}_1$, which is her new "there." At the same time b is no longer imaginary but is now $-b$ and the observer $\mathbb{O}_2$ considers it real because they are both "here" in the space $\mathbb{S}_2$.

Next step: if the observer $\mathbb{O}_2$ now decides to return to the space $\mathbb{S}_1$, she must undo the first motion, so that her view of z_2 is multiplied by $-\mathbf{i}$ so that:

$$z_3 = z_2 \cdot (-\mathbf{i}) = -z_1\mathbf{i}^2 = -a\mathbf{i}^2 + b\mathbf{i} = a + b\mathbf{i} = z_1 \tag{5.0.4}$$

It is not surprising that after the round trip of the observer $\mathbb{O}_2$ to the space $\mathbb{S}_2$ her view of the complex number z_1 returns to the one she had before the trip into the space $\mathbb{S}_1$.

A simpler version of this thought experiment is to imagine a family of three: John, Jane, and their son Mike, who owns a cat. They all live in a house with at least two rooms: a living room and a bedroom. At first, they all gather in the living room, which they consider "here" to themselves. The bedroom is empty because none of them inhabits it.

However, when Mike and his cat move into the bedroom, that room becomes "here" for him, while his parents remain "there"in the living room. And from his parents' point of view, Mike is now "there," in the bedroom. Yet they are uncertain about how Mike treats the cat. Is he treating the cat lovingly or is he being cruel?

When Jane decides to join Mike in the bedroom, it becomes "here" for both of them. She now knows the cat's condition. John, on the other hand, who has remained "there" in the living room, has no way of knowing whether the cat is alive or dead in the bedroom until he too decides to go to Jane and Mike to find out the truth. At that moment, the bedroom becomes "here" while the living room is the new "there" for the whole family.

These two exercises are key to understanding complex numbers and quantum spaces in which elementary particles reside. Simply put, both spaces $\mathbb{S}_1$ and $\mathbb{S}_2$ are equally real, but they are observer-dependent and exchange their perceived nature as "real" or "imaginary" depending on what is "here" or "there" for each observer. And as we have seen, two observers in different spaces can get different results for the same complex number.

And perhaps this explains Bombelli's "wild thought." His whole idea might have been to take a roundabout way to store numbers in a theoretical space, using the terms "più di meno" (pdm) and "meno di meno" (mdm). In his time, he could explain their existence only by this "wild thought."

6. Elementary particles exist in independent and real spaces

The hypothesis that demonstrates the usefulness of the mathematical concepts of "here" and "there" in dual-spaces explains the existence of elementary particles contained in quantum spaces.

Despite being dimensionless, these particles exhibit various physical properties such as mass, electric charge, and spin. While mass and electric charge are familiar concepts in classical physics, spin is a purely quantum mechanical property that represents the particle's angular momentum. However, unlike a spinning top, a particle's spin does not signify literal rotation. Instead, it manifests as a magnetic moment and influences the particle's quantum behavior.

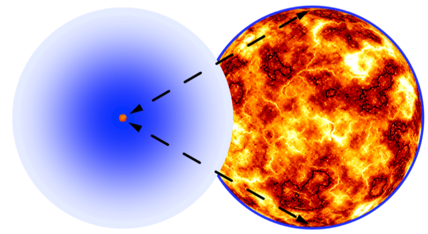

FIGURE 4. The real elementary particle.

Observers looking at a particle "from here," i.e. from our universe, or "from outside" the particle, perceive it as a dimensionless mathematical point. Moreover, what we perceive as the particle's field is actually its boundary, which extends across the entire space of our universe.

This dual perspective applies not only to elementary particles, but also to other spaces, regardless of their actual size. For this reason it is impossible to estimate the size of particles when we look at them "from the outside." As one might reasonably expect, it is equally impossible for a real observer to inhabit the space of a particle or other spaces, at least for the moment.

However, theoretical observers looking at a particle "from the inside" would perceive a physical space with finite non-zero dimensions. From a physical point of view, "absolutely anything" could exist in an orthogonal space, regardless of its size or complexity.

In simpler terms, orthogonal spaces are distinct and independent from the spacetime of the universe. They are physically embedded in the space of the universe, yet simultaneously are external from it.

The concept of orthogonality of spaces, while physical, does not represent a rotation into a spatial dimension. Rather, it is a "relativistic rotation" into a dimension corresponding to the relative velocity between the two frames. Consequently, we do not even know whether we can call it a "dimension" in the sense of the traditional physical definition of the term.

This dimension could also be interpreted as a novel degree of freedom for the space-time frames themselves and not for the objects they contain. Thus, we must be very careful when trying to understand the subtle difference between a relativistic rotation of a space-time frame relative to another frame and a physical rotation of an object within a reference frame.

Moreover, the ability of one independent space to rotate relative to another implies that space and time may not be the most fundamental aspects of reality.

For more details on orthogonal spaces, one can see the article "The Multispace Diagram vs. the Minkowski Diagram." (https://convergetics.org/2019/03/the-multispace-diagram/)

In summary, when a particle is viewed from our perspective—“from here” or “from the outside” its core appears dimensionless and its field—which is its boundary—is infinite. In contrast, when viewed “from there”— or “from within”—the core may appear having any measurable dimensions.

Penrose is right when he talks about the crucial role of complex numbers in quantum theory. He just did not imagine that Bombelli’s “wild thought” could describe an independent and real space.

With this explanation, we can address his paradox and accurately describe reality:

> “*The role of complex numbers in quantum theory is in fact quite crucial. The key point is that the quantum spaces containing elementary particles are real and not imaginary. To understand them properly, we must first understand that our universe is an overarching four-dimensional spacetime manifold, as Einstein believed, but with a secret twist. Quantum spaces are real spaces that contain elementary particles and are embedded in, but at the same time distinct from, the space of the universe. For this reason, elementary particles are dimensionless for us, and we can only perceive their fields.*”

References

[1] Roger Penrose. Twistor geometry of light rays. *Classical and Quantum Gravity*, 14(1A): A299, jan 1997. doi: 10.1088/0264-9381/14/1A/023. URL https://dx.doi.org/10.1088/0264-9381/14/1A/023.

[2] Rafael Bombelli. *L’Algebra*. Mathematica italiana, 1572. URL https://matematicaitaliana.sns.it/opere/9/.

[3] René Descartes *La Géométrie*. Britannica. URL https://www.britannica.com/topic/La-Geometrie.

[4] Carl Friedrich Gauss. *Theoria Residiorum Biquadrati- corum, Commentatio secunda.* In: Goettingen Werke.Bd. 2 (1863), p. 177. URL https://www.gutenberg.org/files/44730/44730-h/44730-h.htm.

[5] William Rowan Hamilton. *Theory of Conjugate Functions, or Algebraic Couples; With a Preliminary and Elementary Essay on Algebra as The Science of Pure Time*, volume vol. 17, part 1. Transactions of the Royal Irish Academy, 1837. URL https://www.maths.tcd.ie/pub/HistMath/People/Hamilton/PureTime/PureTime.pdf.

[6] Peirce, Benjamin. *On the Uses and Transformations of Linear Algebra.*, volume vol. 17, part 1. Proceedings of the American Academy of Arts and Sciences, 1874, 10, 395–400. doi: 10.2307/20021428. URL https://www.jstor.org/stable/20021428.

[7] Pertti Lounesto. *Clifford Algebras and Spinors.* Cambridge University Press, 2 edition, 2001. ISBN 978-0521005517.

[8] *Observer’s Mathematics Approach to Quantum Mechanics.* URL http://staff.www.ltu.se/~norbert/JNMP-Conference-2013/khots.pdf.

[9] *The American Heritage Dictionary of the English Language,*. Houghton Mifflin Harcourt; Thumb Indexed edition, 4 edition, 2000. ISBN 978-0395825174. URL https://www.ahdictionary.com/word/search.html?q=real+number.

[10] Ivan I. Kovalevsky, J.; Mueller."introduction". reference frames. *Astrophysics and Space Science Library.*, 154:1–12, 1989. doi: 10.1007/978-94-009-0933-5_1.

[11] *The Tao of Physics: An Exploration of the Parallels between Modern Physics and Eastern Mysticism Paperback*. Shambhala, 5 edition, 2010. ISBN 978-1590308356. URL https://www.amazon.ca/Tao-Physics-Exploration-Parallels-Mysticism/dp/1590308352/.

Convergetics Research Center
Email address: gene@convergetics.org